Hands-on Session #2 "Gravitational waves"

1 Background

In this hands-on-session we will focus on the production of GWs due to decaying or forced MHD turbulence, reproducing the results of Roper Pol et al. 2020 (arXiv:1903.08585). The system of MHD equations describing the primordial plasma during the radiation-dominated era is

$$\partial_t \ln \rho = -\frac{4}{3}\big(\mathbf{u}\cdot\nabla\ln\rho + \nabla\cdot\mathbf{u}\big) + \frac{1}{\rho}\big[\boldsymbol{u}\cdot\big(\boldsymbol{J}\times\boldsymbol{B}\big) + \eta J^2\big], \tag{1}$$

$$\partial_t \mathbf{u} = -\mathbf{u}\cdot\nabla\mathbf{u} + \frac{\boldsymbol{u}}{3}\big(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho\big) - \frac{1}{4}\nabla\ln\rho + \frac{3}{4\rho}\boldsymbol{J}\times\boldsymbol{B} + \frac{2}{\rho}\boldsymbol{\nabla}\cdot\big(\rho\nu\boldsymbol{S}\big) - \frac{\boldsymbol{u}}{\rho}\big[\boldsymbol{u}\cdot\big(\boldsymbol{J}\times\boldsymbol{B}\big) + \eta J^2\big], \tag{2}$$

$$\partial_t \boldsymbol{A} = \boldsymbol{u}\times\boldsymbol{B} - \eta\mu_0\boldsymbol{J}\,. \tag{3}$$

as used in Roper Pol et al. 2020 (arXiv:1903.08585). See Brandenburg et al. 1996 (arXiv:astro-ph/9602031) and Roper Pol & Midiri 2025 (arXiv:2501.05732).
The resulting GWs from MHD turbulence are computed solving the wave equation

$$\big(\partial_t^2 - c^2\nabla^2\big)h_{ij} = \frac{16\pi G}{ac^2}T_{ij}^{\rm TT}\,, \tag{4}$$

where $T_{ij}^{\rm TT} = (u_i u_j)^{\rm TT} - (B_i B_j)^{\rm TT}$ is the traceless and transverse projection of the stress-energy tensor sourcing GWs and a is the scale factor determining the expansion of the Universe. You can find details on the numerical implementation of the GW equation used in Pencil Code in Roper Pol et al. 2018 (arXiv:1807.05479).

2 Exercises

In this hands-on-session, the objective is to reproduce the results from Roper Pol et al. 2020 (arXiv:1903.08585) using the GW_turbulence code to read data from Pencil Code simulations and plot the resulting time series and spectra, as we already did in the hands-on-session 4. Note that alternatively, you can also use the Python routines installed within Pencil Code (see the PC manual). In the future, the public package CosmoGW (see Read the Docs and lecture at last Pencil Code school if interested in learning more!) will incorporate these reading routines for Pencil Code and other codes (stay tuned!).

(a) Use the Jupyter notebook `results_PRD_1903_08585.ipynb` stored in the directory PRD_1903_08585 of the GW_turbulence repository to reproduce the results of the paper.
(b) Choose your favorite run of the directory and plot the kinetic, magnetic, and GW spectra at a given time. Comment on the resulting spectra. How do they change at different times?

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(c) Choose your favorite run of the directory and plot the total kinetic, magnetic, and GW energy density (averaged over the volume) as a function of time. How do the different energy contributions evolve? Compare the "ini" and the "hel" runs. What are the main differences?

(d) From the plot (already prepared in the notebook) of the resulting GW spectrum of the "ini" runs compared to LISA's sensitivity, what can we say about the resulting GW spectrum? What are the different ranges of frequency dependence found? What do we expect to find at smaller and larger frequencies, the same or a different scaling? If different, what would be the expected scaling?

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