Handout 4: dynamos

Dynamos convert kinetic energy into magnetic energy. Significant amounts of kinetic energy are being released with the beginning of cosmological structure formation. The resulting kinetic energy is in the form of turbulent motions on all scales between those of galaxy clusters, galaxies, stars, and planets.

1 History

Larmor (1919) conceived the idea that kinetic energy can be the reason for the existence of the magnetic field in sunspots. Cowling (1933) showed mathematically that a simple dipole field cannot be sustained by any fluid motion. It was not until 1958 that it was clear that dynamos can exist (Herzenberg, 1958; Backus, 1958). While Parker (1955) did already understand the basics of a solar-type dynamo, this did not stop Chandrasekhar (1956) from looking for alternatives. Even Parker himself continued to work in many other fields (solar wind, interplanetary magnetic fields, reconnection, solar flares, cosmic rays) until he returned to dynamos after Moffatt (1970a,a) called attention to the works of Steenbeck et al. (1966) and Steenbeck & Krause (1969,?), and Roberts & Stix (1971) translated them from German.

2 Types of dynamos

There are different types of dynamos. A precise distinction can easily be problematic. The following subsections distinguish contrasting pairs of dynamo types, giving an idea about the wealth of different possibilities.

2.1 Selfexcited versus non-selfexcited dynamos

In different communities, the term dynamo can mean different things. In astrophysics, we speak about selfexcited dynamos, where no external magnetic field is required. The same distinction also applies to technical dynamos, where selfexcited dynamos are those without any permanent magnets.

2.2 Large-scale and small-scale dynamos

Large-scale dynamos generate magnetic fields on scales larger than the typical scale of the underlying motions. One can then define an average, under which significant magnetic energy still survives. Large-scale dynamos can therefore also be described as mean-field dynamos, at least in principle.

Astrophysically, small-scale dynamo action is very generic. Its correct theoretical description goes back to Kazantsev (1968). Whenever the magnetic Reynolds number is large enough [$\mathrm{Re}_M \equiv u_{\mathrm{rms}}/\eta k_{\mathrm{f}} > O(100)$], a magnetic field grows to equipartition strength with the kinetic energy. Thus, non-magnetic turbulence does not exist in much of the contemporary universe.

2.3 Helical and nonhelical dynamos

When the flow has kinetic helicity, i.e., $\langle\boldsymbol{\omega}\cdot\boldsymbol{u}\rangle/neq0$, where $\boldsymbol{\omega}=\boldsymbol{\nabla}\times\boldsymbol{u}$ is the vorticity, large-scale dynamos can be excited. They are driven by what is called an α effect, which means that the mean electromotive force $\langle\boldsymbol{u}\times\boldsymbol{b}\rangle$ from the small-scale velocity $\boldsymbol{u}=\boldsymbol{U}-\overline{\boldsymbol{U}}$ and the small-scale magnetic field $\boldsymbol{b}=\boldsymbol{B}-\overline{\boldsymbol{B}}$ has a component parallel to the magnetic field. The coefficient in front of $\overline{\boldsymbol{B}}$ is called α, which can also be a tensor or, more generally, a tensorial integral kernel. This contribution is called non-diffusive. There is always also a diffusive contribution proportional to the mean current density $\overline{\boldsymbol{J}}$ and the coefficient is the turbulent magnetic diffusivity. Thus, in its simples form, we have

$$\langle \boldsymbol{u} \times \boldsymbol{b} \rangle = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}. \tag{1}$$

However, not all large-scale dynamos are helical. Examples of nonhelical large-scale dynamos are:

- dynamos driven by a negative turbulent magnetic diffusivity (Devlen et al., 2013),
- dynamos driven by a memory effect (Rheinhardt et al., 2014),
- dynamos driven by an incoherent α effect (Vishniac & Brandenburg, 1997),

2.4 Vortical and non-vortical dynamos

Usual turbulence is vortical, i.e., $\boldsymbol{\omega} \neq 0$. Nonvortical turbulence is called acoustic turbulence (Kadomtsev & Petviashili, 1973). Theoretically, dynamo action from acoustic turbulence should be possible (Kazantsev et al., 1985), but their reality in simulations is still not conclusive; see Achikanath Chirakkara et al. (2021) for possible evidence.

2.5 Statistically stationary and non-stationary dynamos

Many astrophysically relevant flows are statistically non-stationary. Examples are decaying turbulence (Lecture 3) and collapsing flows. Detecting dynamos in non-stationary flows is problematic, because there is no clean exponential growth that is otherwise expected. It may help to transform such collapsing systems into a non-collapsing one (Brandenburg & Ntormousi, 2025).

Interestingly, for stationary flows, one can show that dynamos do not exist when the magnetic diffusivity η vanishes; see Moffatt & Proctor (1985). In the absence of magnetic diffusivity (not just the limit $\eta \to 0$), one can write $\boldsymbol{B}$ in terms of Euler potentials $\alpha_{\rm E}$ and $\beta_{\rm E}$ as

$$\boldsymbol{B} = \boldsymbol{\nabla} \alpha_{\rm E} \times \boldsymbol{\nabla} \beta_{\rm E}. \tag{2}$$

In that case,

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B}) \quad \text{is equivalent to} \quad \frac{{\rm D}\alpha_{\rm E}}{{\rm D}t} = 0 \quad \text{and} \quad \frac{{\rm D}\beta_{\rm E}}{{\rm D}t} = 0. \tag{3}$$

However, dynamos with Euler potentials have never been found (Brandenburg, 2010).

2.6 Fast and slow dynamos

Fast dynamos are those that maintain a finite growth rate in the limit of large ${\rm Re}_M$. For slow dynamos, the growth rate becomes zero in the limit of large ${\rm Re}_M$. In astrophysics, ${\rm Re}_M$ is usually huge, so slow dynamos are of limited interest.

Generally, all dynamos whose flows are integrable are slow. ABC flows, by contrast, are not integrable and may be fast (Galloway & Frisch, 1986). But turbulence is usually enough to make dynamos fast.

2.7 Further subdivision of large-scale dynamos

2.7.1 Dipolar versus quadrupolar

Because of the hemispheric variation of the α effect (negative in the north and positive in the south), the eigenmodes of a linear dynamo ($\boldsymbol{U}$ is unaffected by $\boldsymbol{B}$) cannot be pure multipoles proportional to $Y_{\ell m}(\theta, \phi)$. All the even ℓ couple and all the odd ℓ couple. A dipole has $\ell = 1$, but it couples to $\ell = 3$, 5, etc. Likewise, a quadrupole has $\ell = 2$, but it couples to $\ell = 4$, 6, etc. But even and odd modes do not couple. Only in the nonlinear regime, there can be mixed modes (Brandenburg et al., 1989).

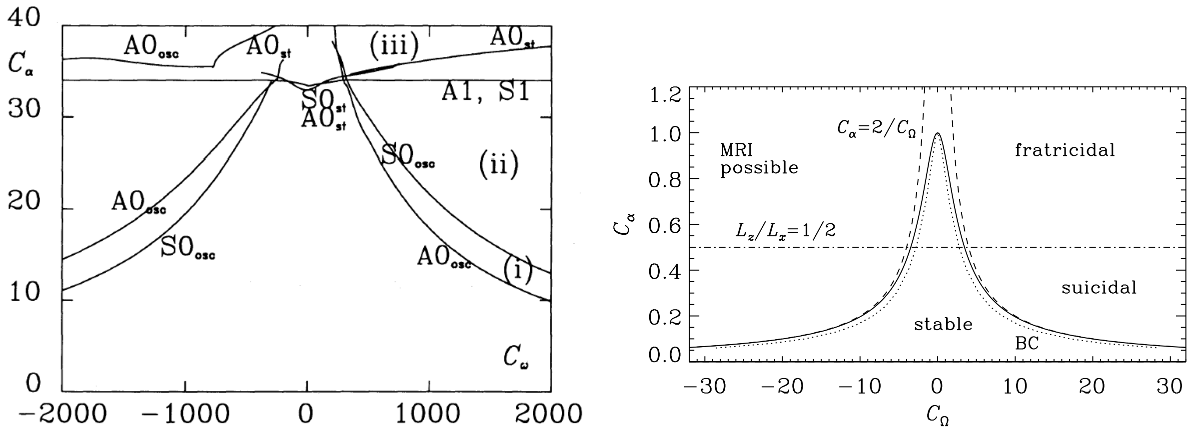

Figure 1: Left: Rädler diagram for an $\alpha^2\Omega$ dynamo in a sphere showing critical values of C_α versus C_ω for a dynamo in a spherical shell. Note that near $C_\omega = 200$ the nonaxisymmetric modes S1 and A1 are more easily excited than the axisymmetric modes S0 and A0. Here, C_ω (called C_Ω in the rest of this review) is defined such that it is positive when $\partial\Omega/\partial r$ is negative, and vice versa. Adapted from Brandenburg et al. (1989). Right: Rädler diagram for the $\alpha^2\Omega$ dynamo with z extent (solid line) and the α^2 dynamo with x extent in a domain with $L_z/L_x = 1/2$ (horizontal dash-dotted line). The onset location in the pure $\alpha\Omega$ approximation ($C_\alpha C_\Omega = 2$) is shown as dashed lines. The case with the vertical field boundary condition is shown as the dotted line and is marked BC.

2.7.2 Axisymmetric and nonaxisymmetric dynamos

An axisymmetric magnetic field has $m = 0$ and a nonaxisymmetric magnetic field has $m \neq 0$. In the linear regime, dynamos with different m do not couple. Usually, axisymmetric magnetic fields are easier to excite than nonaxisymmetric ones. It is therefore not easy to explain nonaxisymmetric magnetic fields in galaxies such as M81. Most spiral galaxies do have axisymmetric magnetic fields.

2.7.3 α^2 and $\alpha\Omega$ dynamos

The presence of shear modifies the dynamo. This is called the Ω effect. For axisymmetric magnetic fields, shear tends to amplifies the field. A dynamo based on the α effect is called an $\alpha\Omega$ dynamo. For nonaxisymmetric magnetic fields, however, shear brings oppositely directed fields close together, so such fields are not amplified by shear.

When there is no shear, a dynamo based on the α effect is called an α^2 dynamo. To distinguish the case where the toroidal magnetic field is purely due to the Ω effect from those where also the α effect plays a role, one sometimes talks about $\alpha^2\Omega$ dynamos, where both the α effect and the Ω effect contribute to replenishing the toroidal field.

The strengths of α effect and Ω effect are characterized by two dynamo numbers

$$C_\alpha = \alpha_0 R/\eta_{\rm t}, \quad C_\Omega = \Delta\Omega R^2/\eta_{\rm t}, \tag{4}$$

where α_0 is the maximum value of α, $\Delta\Omega$ is a measure of the strength of the differential rotation (or shear), and R is the radius of the star or galaxy. A plot of the critical values of C_α versus C_ω is called a Rädler diagram.

With the numerical sample setup for α^2 dynamos, we can address a few questions.

- What are the critical dynamo numbers for dipoles and quadrupoles? Which one is easier to excite?
- What is the difference between dynamos with symmetric and antisymmetric α effect?

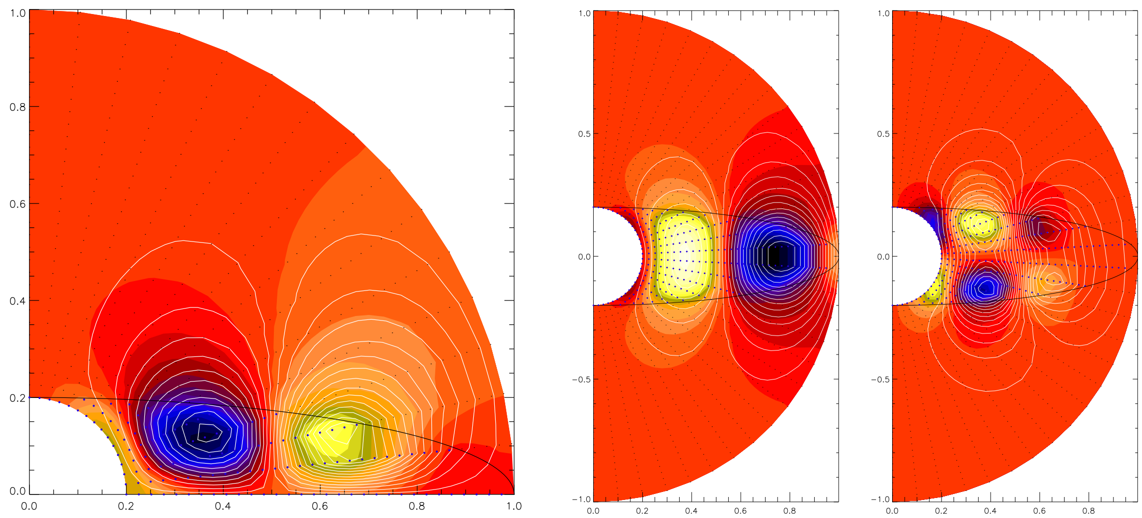

Figure 2: Dynamo solution with an α that is symmetric (left) and antisymmetric (right) between north and south obtained with versions of `https://norlx65.nordita.org/~brandenb/teach/PencilCode/COSMOMAG2026/4_dynamos/material/both-hemispheres/`.

3 Roberts flow dynamos

Roberts (1972) proposed four flow fields to study dynamo action. It is a kinematic flow, i.e., the Navier-Stokes equation is not solved. The velocity depends only on x and y, but all three components of the velocity vector $\boldsymbol{u}$ are non-vanishing. Roberts flow I can be written as $\boldsymbol{u} = k_\mathrm{f}\varphi\hat{\boldsymbol{z}} + \boldsymbol{\nabla}\times(\varphi\hat{\boldsymbol{z}})$, where $\varphi = (v_0/k_0)\,\sin k_0 x \sin k_0$ with v_0 and k_0 being constants. The wavenumber k_f is treated as a free parameter. More explicitly, we can write for all four flows:

$$\boldsymbol{u}_\mathrm{I}(x,y) = \begin{pmatrix} +v_0 \sin k_0 x \cos k_0 y \\ -v_0 \cos k_0 x \sin k_0 y \\ w_0 \sin k_0 x \sin k_0 y \end{pmatrix}, \qquad \boldsymbol{u}_\mathrm{II}(x,y) = \begin{pmatrix} +v_0 \sin k_0 x \cos k_0 y \\ -v_0 \cos k_0 x \sin k_0 y \\ w_0 \cos k_0 x \cos k_0 y \end{pmatrix}. \tag{5}$$

$$\boldsymbol{u}_\mathrm{III}(x,y) = \begin{pmatrix} +v_0 \sin k_0 x \cos k_0 y \\ -v_0 \cos k_0 x \sin k_0 y \\ w_0/2(\cos 2k_0 x + \cos 2k_0 y) \end{pmatrix}, \qquad \boldsymbol{u}_\mathrm{IV}(x,y) = \begin{pmatrix} +v_0 \sin k_0 x \cos k_0 y \\ -v_0 \cos k_0 x \sin k_0 y \\ w_0 \sin k_0 x \end{pmatrix}. \tag{6}$$

where $w_0 = (k_\mathrm{f}/k_0)\,v_0$ has been used. Figure 3 shows the dependence of the growth rate on Re_M.

4 Applications

Stellar and galactic dynamos have large-scale fields and they are probably all of $\alpha^2\Omega$ type. Elliptical galaxies may host small-scale dynamos Moss & Shukurov (1994). The magnetic field in galaxy clusters is probably also due to a small-scale dynamo.

Dynamos could also operate during first-order electroweak or QCD phase transitions, but the problem is that it is unclear whether sufficient vorticity is being produced. There is so far no simulation that shows dynamo action from such flows.

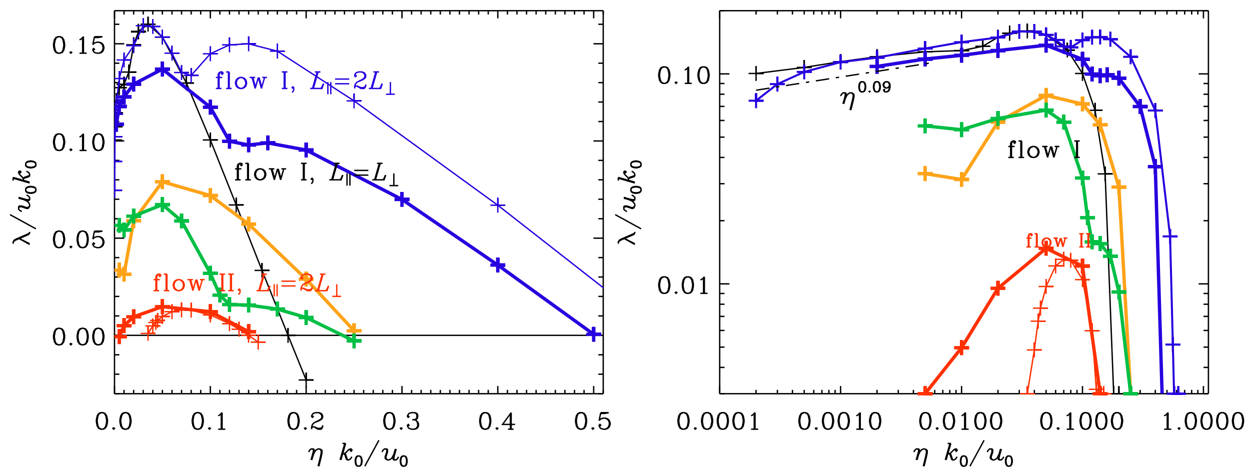

Figure 3: Dependence of the normalized growth rate on the normalized magnetic diffusivity for Roberts flow I with $L_\parallel = L_\perp$ (black lines) and $L_\parallel = 2L_\perp$ (blue lines), as well as flow II for $L_\parallel = 2L_\perp$ (red lines).

References

Achikanath Chirakkara, R., Federrath, C., Trivedi, P., & Banerjee, R., "Efficient Highly Subsonic Turbulent Dynamo and Growth of Primordial Magnetic Fields," *Phys. Rev. Lett.* **126**, 091103 (2021).

Backus, G. E., "A class of self-sustaining dissipative spherical dynamos," *Ann. Phys.* **4**, 372–447 (1958).

Brandenburg, A., "Magnetic field evolution in simulations with Euler potentials," *Month. Not. Roy. Astron. Soc.* **401**, 347–354 (2010).

Brandenburg, A., & Ntormousi, E., "Magnetic field amplification during a turbulent collapse," *Astrophys. J.* **990**, 223 (2025).

Brandenburg, A., Krause, F., Meinel, R., Moss, D., & Tuominen, I., "The stability of nonlinear dynamos and the limited role of kinematic growth rates," *Astron. Astrophys.* **213**, 411–422 (1989).

Chandrasekhar, S., "Effect of Internal Motions on the Decay of a Magnetic Field in a Fluid Conductor," *Astrophys. J.* **124**, 244– (1956).

Cowling, T. G., "The magnetic field of sunspots," *Month. Not. Roy. Astron. Soc.* **94**, 39–48 (1933).

Devlen, E., Brandenburg, A., & Mitra, D., "A mean field dynamo from negative eddy diffusivity," *Month. Not. Roy. Astron. Soc.* **432**, 1651–1657 (2013).

Galloway, D., & Frisch, U., "Dynamo action in a family of flows with chaotic streamlines," *Geophys. Astrophys. Fluid Dynam.* **36**, 53–84 (1986).

Herzenberg, A., "Geomagnetic dynamos," *Phil. Trans. Roy. Soc. A* **250**, 543–583 (1958).

Kadomtsev, B. B. & Petviashili, V. I., "Acoustic turbulence," *Sov. Phys. Dokl.* **18**, 115–116 (1973).

Kazantsev, A. P., "Enhancement of a magnetic field by a conducting fluid," *Sov. Phys. JETP* **26**, 1031–1034 (1968).

Kazantsev, A. P., Ruzmaikin, A. A., & Sokoloff, D. D., "Magnetic field transport by an acoustic turbulence-type flow," *Sov. Phys. JETP* **88**, 487–494 (1985).

LARMOR, J., "How could a rotating body such as the Sun become a magnet," *Rep. Brit. Assoc. Adv. Sci.* **87**, 159–160 (1919).

Moffatt, H. K., "Turbulent dynamo action at low magnetic Reynolds number," *J. Fluid Mech.* **41**, 435–452 (1970).

Moffatt, H. K., "Dynamo action associated with random inertial waves in a rotating conducting fluid," *J. Fluid Mech.* **44**, 705–719 (1970b).

Moffatt, H. K., & Proctor, M. R. E., "Topological constraints associated with fast dynamo action," *J. Fluid Mech.* **154**, 493–507 (1985).

Moss, D., & Shukurov, A., "Turbulence and magnetic fields in elliptical galaxies," *Month. Not. Roy. Astron. Soc.* **279**, 229–239 (1994).

Parker, E. N., "Hydromagnetic dynamo models," *Astrophys. J.* **122**, 293–314 (1955).

Rheinhardt, M., Devlen, E., Rädler, K.-H., & Brandenburg, A., "Mean-field dynamo action from delayed transport," *Month. Not. Roy. Astron. Soc.* **441**, 116–126 (2014).

Roberts, G. O., "Dynamo action of fluid motions with two-dimensional periodicity," *Phil. Trans. Roy. Soc. A* **271**, 411–454 (1972).

Roberts, P. H. & Stix, M. *The turbulent dynamo.* Tech. Note 60, NCAR, Boulder, Colorado (1971).

Steenbeck, M., Krause, F., & Rädler, K.-H., "Berechnung der mittleren Lorentz-Feldstärke $\overline{\boldsymbol{v} \times \boldsymbol{B}}$ für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflußter Bewegung," *Z. Naturforsch.* **21a**, 369–376 (1966).

Steenbeck, M., & Krause, F., "Zur Dynamotheorie stellarer und planetarer Magnetfelder I. Berechnung sonnenähnlicher Wechselfeldgeneratoren," *Astron. Nachr.* **291**, 49–84 (1969).

Steenbeck, M., & Krause, F., "Zur Dynamotheorie stellarer und planetarer Magnetfelder II. Berechnung planetenähnlicher Gleichfeldgeneratoren," *Astron. Nachr.* **291**, 271–286 (1969).

Vishniac, E. T., & Brandenburg, A., "An incoherent $\alpha-\Omega$ dynamo in accretion disks," *Astrophys. J.* **475**, 263–274 (1997).

$Header: /var/cvs/brandenb/tex/teach/COSMOMAG26/4_dynamos/notes.tex,v 1.6 2026/01/15 12:07:34 brandenb Exp $